Quantum Mechanics in one space General Relativity in another

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November 28, 2025
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ICTP
SAIFR

A picture to have in mind

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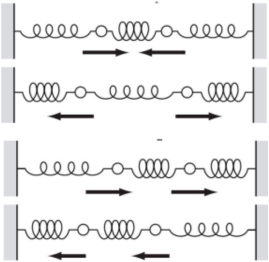
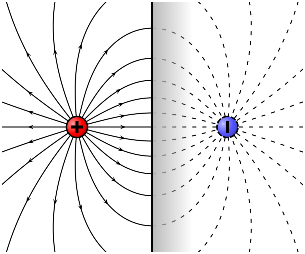


← Normal modes

Image charges →



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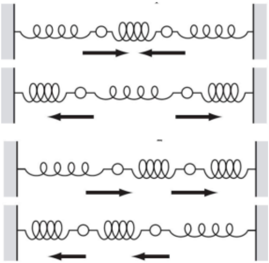

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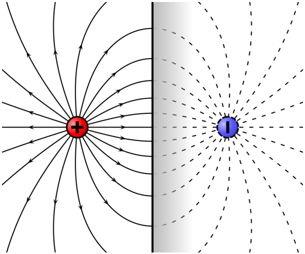

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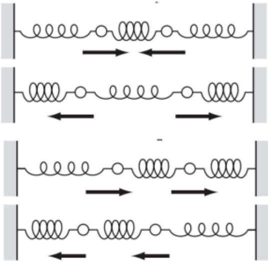

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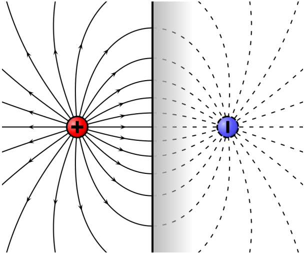

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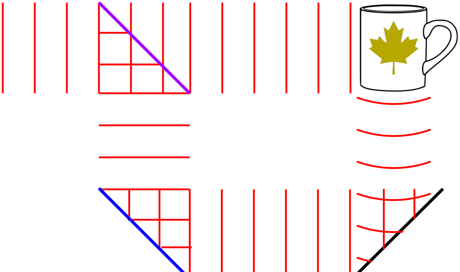

← 3d scene (complicated)

← 2d plate (simple)

Gravity vs other forces

Maxwell's eq'ns determine fields on spacetime.

$$\nabla \cdot \mathbf{E} = 4\pi\rho, \quad \nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$$

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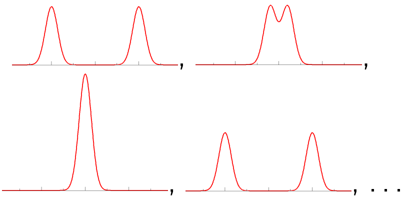

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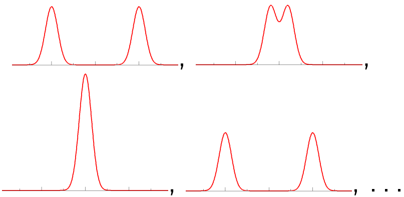

Einstein's equation determines the shape of spacetime

$$g_{\mu\nu} = \mathrm{diag}(-1,1,1,1) \Rightarrow ds^2 = -c^2dt^2 + dx^2 + dy^2 + dz^2$$

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and particles follow geodesics which minimize ds.

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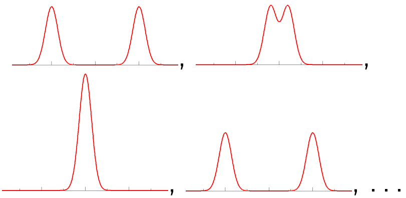

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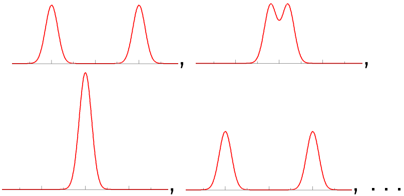

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$$\text{Nonlinear} \leftarrow \quad R_{\mu\nu} - \frac{1}{2}R_{\rho\sigma}g^{\rho\sigma}g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

A simple but profound solution

"Point mass" solution found in [Schwarzschild, 1916] :

$$ds^2 = -\left(1 - \frac{2GM}{c^2 r}\right) c^2 dt^2 + \left(1 - \frac{2GM}{c^2 r}\right)^{-1} dr^2 + r^2 d\Omega^2.$$

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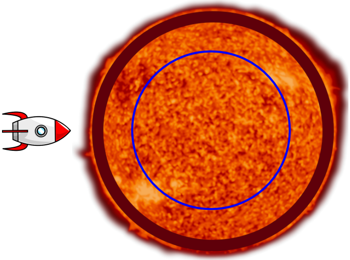

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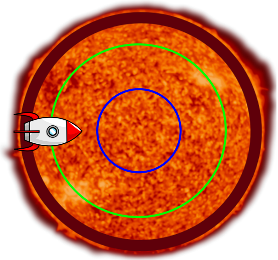

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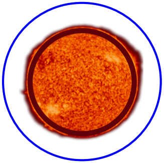

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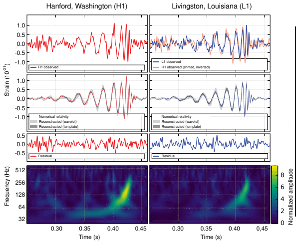

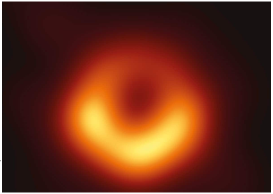

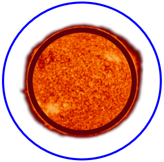

[LIGO, Virgo, 2016]

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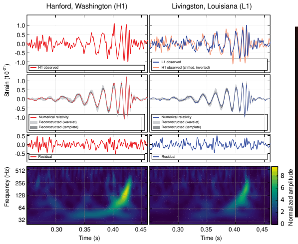

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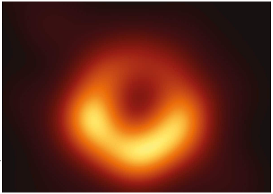

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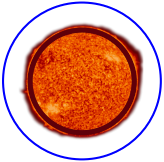
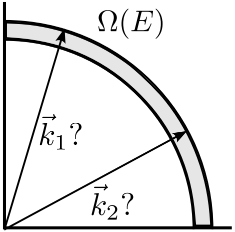


Most efficient packing of mass!

How much information lies inside?

Event horizons also have an interesting entropy [Bekenstein, 1973] .



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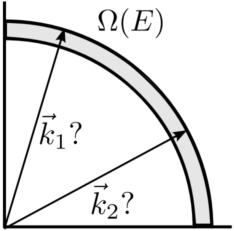

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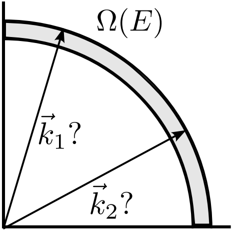

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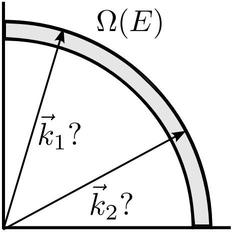

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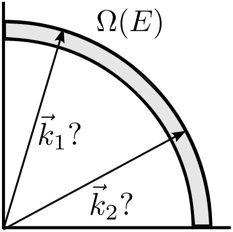

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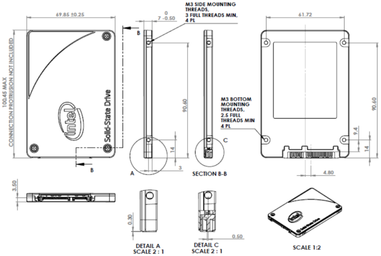

$A = 0.0163\text{m}^2$

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$7.11 \cdot 10^{54}$ TB

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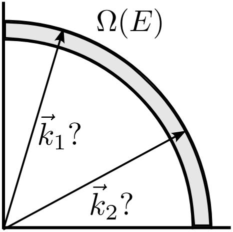

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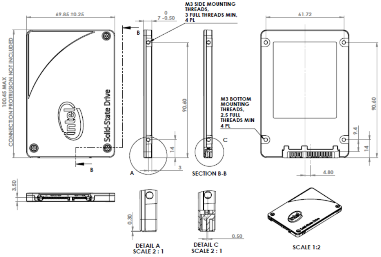

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Thermal state combines $S = k \log \Omega$ and $dE = TdS$:

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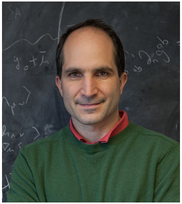
After a black hole has formed, there is a different vacuum killed by "negative frequency" operators.

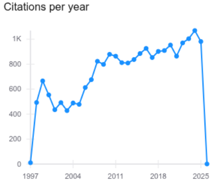
$$b_i = \sum_j \alpha_{ij} a_j + \beta_{ij} a^\dagger_j, \quad N_i = {}_a\langle 0 | b^\dagger_i b_i | 0 \rangle_a = \sum_j |\beta_{ij}|^2.$$

Coefficients found by projecting ingoing wave with constant $v = ct + r$ onto outgoing wave with constant $u = ct - r$ [Hawking, 1975] .

$$u = -\frac{4GM}{c^2} \log(v - v_0) + \cdots \Rightarrow kT = \frac{\hbar c^3}{8\pi GM}$$

A well defined holographic duality





[Maldacena; hep-th/9711200]

Quantum Field Theory $\longleftrightarrow$ Quantum Gravity

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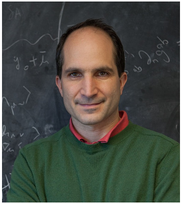

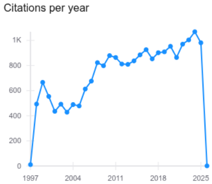

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Confornally invariant Quantum Field Theory	$\longleftrightarrow$	Asymptotically Anti-de Sitter Quantum Gravity

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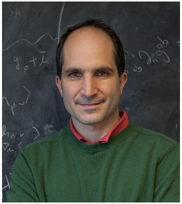

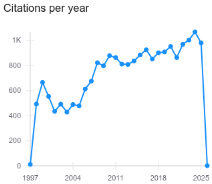

[Maldacena; hep-th/9711200]

Conformally invariant Quantum Field Theory	$\longleftrightarrow$	Asymptotically Anti-de Sitter Quantum Gravity

This AdS/CFT correspondence (discovered using string theory) has implications for the following questions.

1. How to make the extra dimensions of string theory compact?
2. What does string theory predict non-perturbatively?
3. How to treat strings (even perturbatively) when the non-compact space is curved?
4. What changes when a QFT becomes strongly coupled?

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$SO(d+1,1)$

$\Lambda = 0$, Minkowski
$SO(d,1) \times$ Translations

$\Lambda < 0$, Anti-de Sitter
$SO(d,2)$

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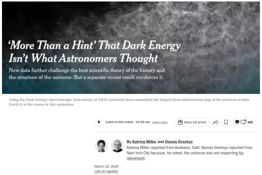
'More Than a Hint' That Dark Energy Isn't What Astronomers Thought

New data further challenge the best scientific theory of the history and the structure of the universe. But a separate recent result reinforces it.

Using the Dark Energy Spectroscopic Instrument, or DESI, scientists have assembled the largest three-dimensional map of the universe to date. Earth is at the center in this animation.

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By Katrina Miller and Dennis Overbye
Katrina Miller reported from Anaheim, Calif. Dennis Overbye reported from New York City because, he noted, the universe was not respecting his retirement.

March 19, 2025
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[DESI, 2024]


Dark energy: mysterious cosmic force appears to be weakening, say scientists

Findings could open up possibility the universe will end in a reverse big bang or 'big crunch', say cosmologists

Explainer: Is dark energy destined to dominate the universe?

Hannah Devlin *Science correspondent*
Wed 19 Mar 2025 22.00 GMT
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Long exposure shows trails of stars above Kitt Peak National Observatory, where astronomers are studying the nature of dark energy. Photograph: AP

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Rotations	$x'_\mu = \Lambda_\mu{}^\nu x_\nu$	$\binom{d}{2}$
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Special	$x'_\mu = \frac{x_\mu - b_\mu x^2}{1 - 2b\cdot x + b^2 x^2}$	d

$SO(d,2)$ symmetric QFT with no preferred scale.

Realized in nature in phase transitions with $\xi = \infty$.

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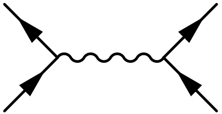

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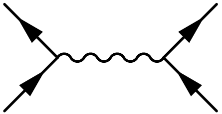

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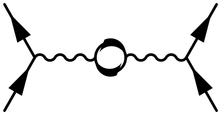

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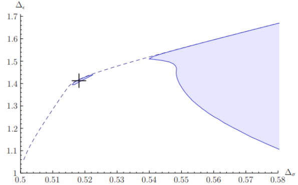
[Kos, Poland, Simmons-Duffin; 1406.4858]



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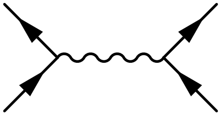

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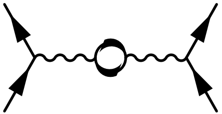

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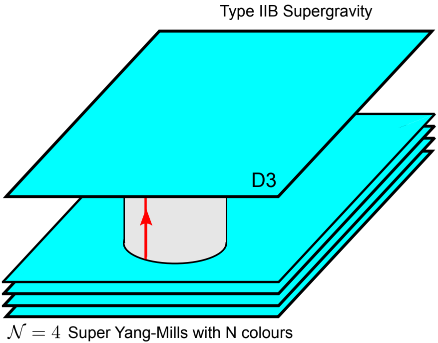

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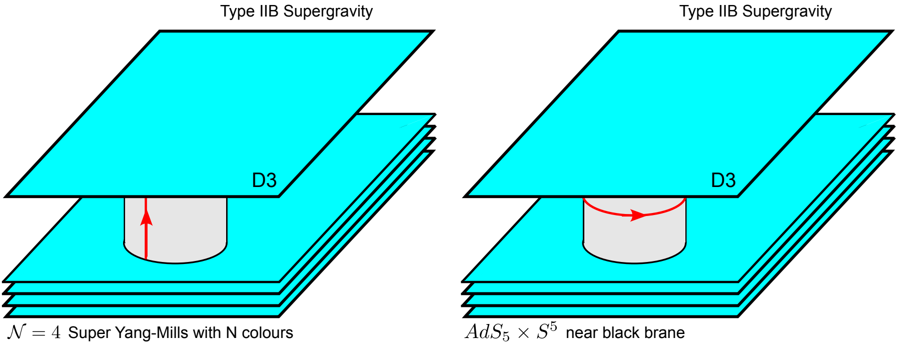

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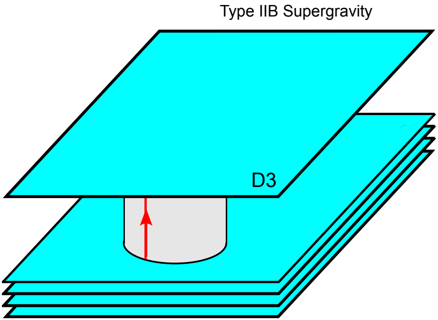
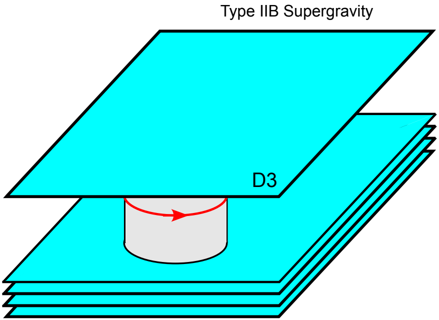


$\mathcal{N} = 4$ Super Yang-Mills with N colours



$AdS_5 \times S^5$ near black brane

Type IIB string theory on $AdS_5 \times S^5$ is equivalent to a CFT labelled by N and g_{YM} [Maldacena; hep-th/9711200] .

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$$A = \sum_{top} \int DXDg \, e^{-S-\lambda S_{top}} \ldots$$

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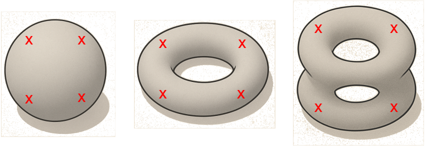

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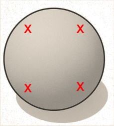

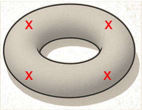

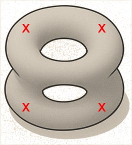

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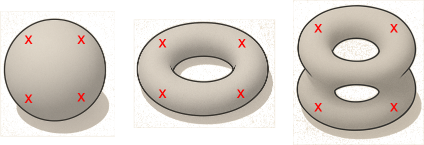

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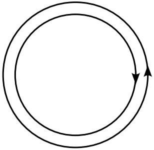

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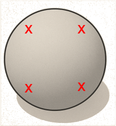

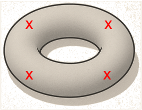

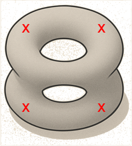

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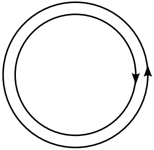

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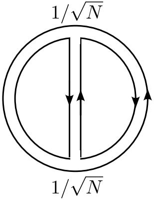

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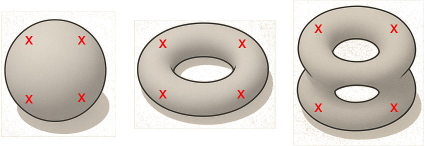

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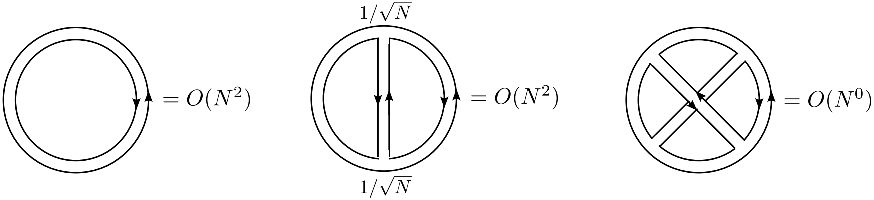

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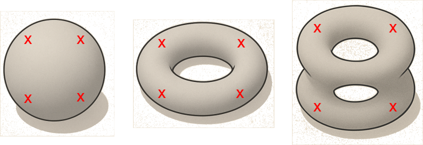

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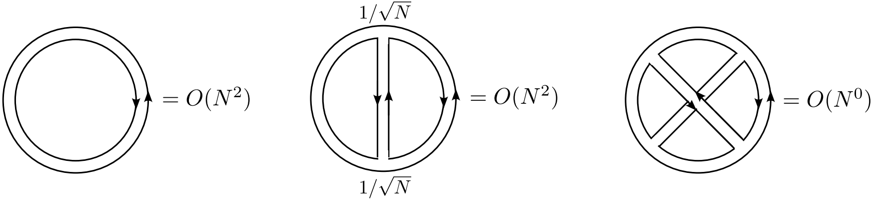

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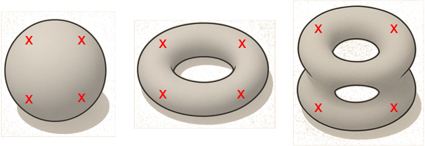

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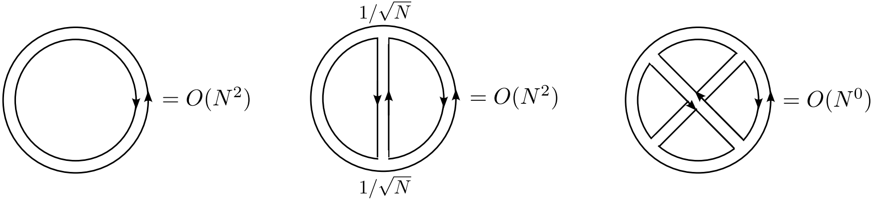

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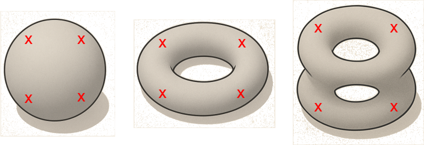

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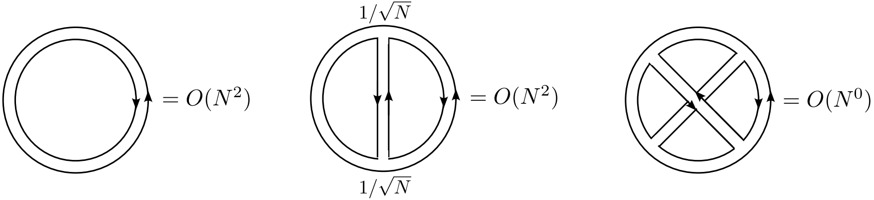

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Calculations in AdS are most tractable with $L/l_p \ll 1$, $L/l_s \ll 1$ (large N, large λ)!

Some basic checks

Asymptotic density of states $S = \log \Omega$ can be computed by dimensional analysis in a CFT.

$$E \propto VT^{d+1}, \quad S \propto VT^{d} \quad \Rightarrow \quad S \propto E^{\frac{d}{d+1}}$$

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Some gauge theory phases at lower energy are harder to study without the correspondence [Witten; hep-th/9803131] .

Beyond counting operators

Consider low energy (or really dimension) operators.

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The same techniques work for another holographic setup with $\mathcal{N} = 2$ SUSY [CB, Chester, Ferrero; 2305.01016] . Result matches open strings for $L \to \infty$.

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Bound states in QCD obey wave equations which have an AdS_5 interpretation [Forshaw, Sandapen; 2501.00526] .

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$$\Delta^{\mu\nu} \equiv g^{\mu\nu} + u^\mu u^\nu$$

Can also model $\mathcal{N} = 4$ Super Yang-Mills when deviations from thermal equilibrium are slowly varrying [Kovtun, Son, Starinets; hep-th/0405241] .

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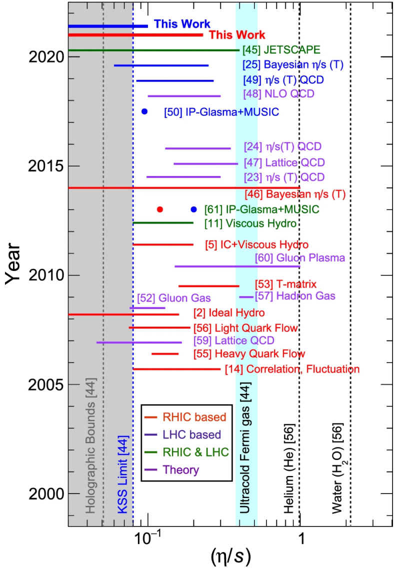

[Gonzalez, Basu, Guevara, Marin, Pan, Pruneau; 2012.10542]

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Thanks for your attention!